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UNSTEADY VIBRATIONS OF AN ELASTIC MEDIUM BOUNDED BY TWO ECCENTRIC SPHERICAL SURFACES†

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The axisymmetric problem of the propagation of unsteady waves in an elastic medium bounded by spherical surfaces with offset centres is considered. The problem of the propagation of waves in continuous media bounded by surfaces of different coordinate systems has been investigated fairly fully mainly in the steady-state formulation [1]. Some unsteady problems for a half-space with spherical inclusions were investigated in [2, 3].

1. FORMULATION OF THE PROBLEM

Suppose a linearly elastic uniform isotropic medium is bounded by two eccentric spherical surfaces. The radius of the external surface is R_2 and the radius of the internal sphere is R_1. The distance between the centres of the spheres is $\delta(R_2 > R_1 + \delta)$.

Two spherical systems of coordinates are employed: the origin of the first system $(r_1, \theta_1, \vartheta_1)$ is at the centre of the internal sphere (cavity), while the origin of the second $(r_2, \theta_2, \vartheta_2)$ is at the centre of the external sphere.

Axisymmetrical surface loads

$$\sigma_{r_1r_1}\Big|_{r_1=R_1} = p(\tau,\theta_1), \quad \sigma_{r_1\theta_1}\Big|_{r_1=R_1} = q_1(\tau,\theta_1) \tag{1.1}$$

are applied to the surface of the spherical cavity, or the displacements

$$u_1\Big|_{r_1=R_1} = U(\tau,\theta_1), \quad \upsilon_1\Big|_{r_1=R_1} = V(\tau,\theta_1) \tag{1.2}$$

are given.

There are no stresses on the external sphere

$$\sigma_{r_2r_2}\Big|_{r_2=R_2} = 0, \quad \sigma_{r_2\theta_2}\Big|_{r_2=R_2} = 0 \tag{1.3}$$

or the displacements are zero

$$u_2\Big|_{r_2=R_2} = 0, \quad \upsilon_2\Big|_{r_2=R_2} = 0 \tag{1.4}$$

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Here u_i, υ_i and $\sigma_{\alpha_i\beta_i}$ are the normal and tangential components of the displacement vector and the components of the stress tensor in the systems of coordinates $(r_i, \theta_i, \vartheta_i)$, $i = 1, 2$.

Taking the axial symmetry into account, the perturbed motion of the elastic medium satisfies the following wave equations with respect to the scalar component φ and the non-zero component ψ of the vector potential of the displacements

$$\frac{\partial^2\varphi}{\partial\tau^2} = \Delta_i\varphi, \quad \eta^2\frac{\partial^2\psi}{\partial\tau^2} = \Delta_i\psi - \frac{\psi}{r_i^2\sin^2\theta_i} \tag{1.5}$$

where Δ_i is the Laplace operator in the corresponding system of coordinates.

The initial conditions are assumed to be homogeneous

$$\varphi|_{\tau=0} = \left.\frac{\partial\varphi}{\partial\tau}\right|_{\tau=0} = \psi|_{\tau=0} = \left.\frac{\partial\psi}{\partial\tau}\right|_{\tau=0} = 0 \tag{1.6}$$

The displacements u_i and υ_i and the stresses $\sigma_{\alpha_i\beta_i}$ are related to the potentials φ and ψ by well-known relations of the linear theory of elasticity [4].

In (1.1)–(1.6) and henceforth we have used the following dimensionless parameters (the primes denote dimensional quantities)

$$r_i = \frac{r_i'}{R}, \quad \tau = \frac{c_1 t}{R}, \quad \eta = \frac{c_1}{c_2}, \quad \delta = \frac{\delta'}{R}$$

$$\kappa = \frac{\lambda}{\lambda + 2\mu}, \quad \sigma_{\alpha\beta} = \frac{\sigma'_{\alpha\beta}}{\lambda + 2\mu}, \quad u_i = \frac{u_i'}{R}$$

$$\upsilon_i = \frac{\upsilon_i'}{R}, \quad \varphi = \frac{\varphi'}{R^2}, \quad \psi = \frac{\psi'}{R^2}, \quad R_i = \frac{R_i'}{R}$$

where R is a certain characteristic linear dimension, c_1 and c_2 are the velocities of the extension–compression wave and the shear wave, λ and μ are the elastic Lamé constants, and t is the time.

2. THE METHOD OF SOLUTION

The initial boundary-value problem (1.1)–(1.6) is solved by the method of incomplete separation of variables using an integral Laplace transformation with respect to time τ (s is the transformation parameter and the superscript L corresponds to the transformant).

We will represent the components of the stress–strain state of the medium and the right-hand sides of the boundary conditions (1.1) and (1.2) in transformation space in the form of series in Legendre polynomials $P_n(\cos\theta_i)$ and Gegenbauer polynomials $C_{n-1}^{3/2}(\cos\theta_i)$ $(i = 1, 2)$

$$\left\| \begin{matrix} u_i^L \\ \sigma_{\eta\eta}^L \end{matrix} \right\| = \sum_{n=0}^{\infty} \left\| \begin{matrix} u_{in}^L(r_i, s) \\ \sigma_{\eta\eta n}(r_i, s) \end{matrix} \right\| P_n(\cos\theta_i)$$

$$\left\| \begin{matrix} \upsilon_i^L \\ \sigma_{\eta\theta_i}^L \end{matrix} \right\| = -\sin\theta_i \sum_{n=1}^{\infty} \left\| \begin{matrix} \upsilon_{in}^L(r_i, s) \\ \sigma_{\eta\theta_i n}^L(r_i, s) \end{matrix} \right\| C_{n-1}^{3/2}(\cos\theta_i)$$

$$\left\| \begin{matrix} p_1^L \\ U^L \end{matrix} \right\| = \sum_{n=0}^{\infty} \left\| \begin{matrix} p_{1n}^L(s) \\ U_n^L(s) \end{matrix} \right\| P_n(\cos\theta_1)$$

$$\left\| \begin{matrix} q_1^L \\ V^L \end{matrix} \right\| = -\sin\theta_1 \sum_{n=1}^{\infty} \left\| \begin{matrix} q_{1n}^L(s) \\ V_n^L(s) \end{matrix} \right\| C_{n-1}^{3/2}(\cos\theta_1)$$

The boundary conditions (1.1)–(1.4) with respect to the coefficients of the expansions then take the following form

$$\sigma_{\eta\eta n}^L\Big|_{\eta=R_1} = p_{1n}^L(s), \quad \sigma_{\eta\theta_1 n}^L\Big|_{\eta=R_1} = q_{1n}^L(s) \tag{2.1}$$

$$u_{1n}^L\Big|_{\eta=R_1} = U_n^L(s), \quad \upsilon_{1n}^L\Big|_{\eta=R_1} = V_n^L(s) \tag{2.2}$$

$$\sigma_{r_2r_2n}^L\Big|_{r_2=R_2} = 0, \quad \sigma_{r_2\theta_2 n}^L\Big|_{r_2=R_2} = 0 \tag{2.3}$$

$$u_{2n}^L\Big|_{r_2=R_2} = 0, \quad \upsilon_{2n}^L\Big|_{r_2=R_2} = 0 \tag{2.4}$$

In Laplace transformation space the potentials of the displacements φ and ψ can be written in the form

$$\varphi^L = \sum_{n=0}^{\infty} A_n^L(s)\sqrt{\frac{2s}{\pi r_1}}K_{n+\frac{1}{2}}(r_1 s)P_n(\cos\theta_1) + \sum_{p=0}^{\infty} B_p^L(s)\sqrt{\frac{2\pi s}{r_2}}I_{p+\frac{1}{2}}(r_2 s)P_p(\cos\theta_2)$$

$$\psi^L = -\sin\theta_1 \sum_{n=1}^{\infty} C_n^L(s)\sqrt{\frac{2\eta s}{\pi r_1}}K_{n+\frac{1}{2}}(r_1\eta s)C_{n-1}^{3/2}(\cos\theta_1) -$$

$$-\sin\theta_2 \sum_{p=1}^{\infty} D_p^L(s)\sqrt{\frac{2\pi\eta s}{r_2}}I_{p+\frac{1}{2}}(r_2\eta s)C_{p-1}^{3/2}(\cos\theta_2) \tag{2.5}$$

where $K_\nu(x)$ and $I_\nu(x)$ are modified Bessel functions, and $A_n^L(s)$, $B_p^L(s)$, $C_n^L(s)$, and $D_p^L(s)$ are unknown functions of the parameter s.

Using the addition theorem for the functions $I_{n+\frac{1}{2}}(x)$ and $K_{n+\frac{1}{2}}(x)$ [5], taking into account the expressions for these functions in terms of elementary function [4], and also the relation between the displacements, the stresses and the potential in a spherical system of coordinates, we obtain the following formulae for the coefficients of the series u_{1n}^L, υ_{1n}^L, $\sigma_{r_1r_1n}^L$ and $\sigma_{r_1\theta_1 n}^L$

$$u_{1n}^L(r_1,s) = -r_1^{-n-2}s^{-n}\Bigg[R_{n1}(r_1 s)A_n^L(s)e_{11} + G_{n1}(r_1 s)\sum_{p=0}^{\infty} E_{np}^{(1)}(s)B_p^L(s) +$$

$$+n(n+1)\eta^{-n}R_{n0}(r_1\eta s)C_n^L(s)e_{14} + \eta^{-n}G_{n0}(r_1\eta s)\sum_{p=1}^{\infty} H_{np}^{(1)}(s)D_p^L(s)\Bigg]$$

$$\upsilon_{1n}^L(r_1,s) = r_1^{-n-2}s^{-n}\Bigg[R_{n0}(r_1 s)A_n^L(s)e_{11} + G_{n0}(r_1 s)\sum_{p=0}^{\infty} E_{np}^{(1)}(s)B_p^L(s) +$$

$$+\eta^{-n}R_{n3}(r_1\eta s)C_n^L(s)e_{14} + n^{-1}(n+1)^{-1}\eta^{-n}G_{n3}(r_1\eta s)\sum_{p=1}^{\infty} H_{np}^{(1)}(s)D_p^L(s)\Bigg] \tag{2.6}$$

$$\sigma_{\eta\eta n}^L(r_1,s) = -r_1^{-n-3}s^{-n}\Bigg[Q_{n1}(r_1 s)A_n^L(s)e_{11} + J_{n1}(r_1 s)\sum_{p=0}^{\infty} E_{np}^{(1)}(s)B_p^L(s) +$$

$$+n(n+1)\eta^{-n}Q_{n2}(r_1\eta s)C_n^L(s)e_{14} + \eta^{-n}J_{n2}(r_1\eta s)\sum_{p=1}^{\infty} H_{np}^{(1)}(s)D_p^L(s)\Bigg] \tag{2.7}$$

$$\sigma^L_{\eta\theta_1 n}(r_1,s) = -r_1^{-n-3}s^{-n}\Bigg[Q_{n2}(r_1 s)A^L_n(s)e_{11} + J_{n2}(r_1 s)\sum_{p=0}^{\infty} E^{(1)}_{np}(s)B^L_p(s) +$$

$$+\eta^{-n}Q_{n3}(r_1\eta s)C^L_n(s)e_{14} + J_{n3}(r_1\eta s)n^{-1}(n+1)^{-1}\eta^{-n}\sum_{p=0}^{\infty} H^{(1)}_{np}(s)D^L_p(s)\Bigg]$$

where

$$E^{(1)}_{np}(s) = C^{(1)}_{np}(s)e_{12} - C^{(2)}_{np}(s)e_{13}, \quad H^{(1)}_{np}(s) = S^{(1)}_{np}(s)e_{15} - S^{(2)}_{np}(s)e_{16}$$

$$e_{11} = e^{-\eta s}, \quad e_{12} = e^{\delta s}, \quad e_{13} = e^{-\delta s}, \quad e_{14} = e^{-\eta\eta s}, \quad e_{15} = e^{\delta\eta s}, \quad e_{16} = e^{-\delta\eta s}$$

The polynomials $R_{ni}(s)$ $(i=0,\ 1,\ 3)$ and $Q_{nj}(s)$ $(j=1,\ 2,\ 3)$ used in (2.6) and (2.7) are connected with the representations of modified Bessel functions and are defined in [4], while the functions $G_{nk}(s)$, $J_{nk}(s)$ $(k=1,2,3)$ and $C^{(l)}_{np}(s)$, $S^{(l)}_{np}(s)$ $(l=1,\ 2)$ have the following form

$$G_{nk}(s) = R_{nk}(-s)e^{s} - R_{nk}(s)e^{-s}, \quad J_{nk}(s) = Q_{nk}(-s)e^{s} - Q_{nk}(s)e^{-s}$$

$$C^{(l)}_{np}(s) = \frac{(-1)^p(2n+1)}{2\delta s}\sum_{\sigma=|p-n|}^{p+n} b^{(n0p0)}_{\sigma}\frac{(-1)^{\sigma}}{(\delta s)^{\sigma}}R_{\sigma 0} = [(-1)^l\delta s] \tag{2.8}$$

$$S^{(l)}_{np}(s) = \frac{(-1)^p(2n+1)}{2\delta\eta s}\sum_{\sigma=|p-n|}^{p+n} b^{(n1p1)}_{\sigma}\frac{R_{\sigma 0}[(-1)^l\delta\eta s]}{(\delta\eta s)^{\sigma}}$$

where $b^{(nipi)}_{\sigma}$ $(i=0,\ 1)$ are the Clebsch–Gordan coefficients [5].

The expressions for the coefficients of the series u^L_{2n}, υ^L_{2n}, $\sigma^L_{r_2r_2n}$ and $\sigma^L_{r_2\theta_2 n}$ are similar to (2.6) and (2.7) and can be obtained from the latter if we interchange the coefficients A^L_n and B^L_n, C^L_n and D^L_n, and also replace r_1 by r_2, $E^{(1)}_{np}$ and $H^{(1)}_{np}$ by $E^{(2)}_{np}$ and $H^{(2)}_{np}$, $R_{ni}(r_1\eta_k s)$ by $(-1)^n G_{ni}(r_2\eta_k s)$ and $G_{ni}(r_1\eta s)$ by $R_{ni}(r_2\eta_1 s)$ $(\eta_1=1,\ \eta_2=\eta;\ i=0,\ 1,\ 3)$, $Q_{nj}(r_1\eta_k s)$ by $(-1)^n J_{nj}(r_2\eta_k s)$ and $J_{nj}(r_1\eta s)$ by $Q_{nj}(r_2\eta_k s)$ $(j=1,\ 2,\ 3)$, e_{1m} and e_{2m} by $(m=1\text{–}6)$. Here the last quantities are defined as follows:

$$e_{21} = e_{24} = 1, \quad e_{22} = e^{-(r_2-\delta)s}$$

$$e_{23} = e^{-(r_2+\delta)s}, \quad e_{25} = e^{-(r_2-\delta)\eta s}, \quad e_{26} = e^{-(r_2+\delta)\eta s} \tag{2.9}$$

Substituting the coefficients (2.6) and (2.7) into the boundary conditions (2.1)–(2.4), we obtain an infinite system of linear algebraic equations in the unknown functions $A^L_n(s)$, $B^L_n(s)$, $C^L_n(s)$ and $D^L_n(s)$

$$\mathbf{M}^{(1)}\mathbf{A}\upsilon^2 wzt + \mathbf{N}^{(1)}\mathbf{C}\upsilon w^2 zt + \mathbf{T}^{(1)}_1\mathbf{B}wt + \mathbf{T}^{(2)}_1\mathbf{B}wz^2 t - \mathbf{T}^{(3)}_1\mathbf{B}\upsilon^2 wt + \mathbf{T}^{(4)}_1\mathbf{B}\upsilon^2 wz^2 t +$$
$$+\mathbf{T}^{(5)}_1\mathbf{D}\upsilon z - \mathbf{T}^{(6)}_1\mathbf{D}\upsilon zt^2 - \mathbf{T}^{(7)}_1\mathbf{D}\upsilon w^2 z + \mathbf{T}^{(8)}_1\mathbf{D}w^2\upsilon zt^2 = \mathbf{K}^{(1)}\upsilon wzt$$
$$\mathbf{M}^{(2)}\mathbf{A}\upsilon^2 wzt + \mathbf{N}^{(2)}\mathbf{C}\upsilon w^2 zt + \mathbf{T}^{(1)}_2\mathbf{B}wt - \mathbf{T}^{(2)}_2\mathbf{B}wz^2 t - \mathbf{T}^{(3)}_2\mathbf{B}\upsilon^2 wt + \mathbf{T}^{(4)}_2\mathbf{B}w\upsilon^2 z^2 t +$$
$$+\mathbf{T}^{(5)}_2\mathbf{D}\upsilon z - \mathbf{T}^{(6)}_2\mathbf{D}\upsilon zt^2 - \mathbf{T}^{(7)}_2\mathbf{D}\upsilon w^2 t + \mathbf{T}^{(8)}_2\mathbf{D}w^2\upsilon zt^2 = \mathbf{K}^{(2)}\upsilon wzt$$

$$\mathbf{L}^{(1)}_1\mathbf{B}yzt - \mathbf{L}^{(2)}_1\mathbf{B}x^2 yzt + \mathbf{F}^{(1)}_1\mathbf{A}x^2 yt - \mathbf{F}^{(2)}_1\mathbf{A}x^2 yz^2 t + \mathbf{L}^{(3)}_1\mathbf{D}xzt - \tag{2.10}$$
$$-\mathbf{L}^{(4)}_1\mathbf{D}xy^2 zt + \mathbf{F}^{(3)}_1\mathbf{C}xy^2 z - \mathbf{F}^{(4)}_1\mathbf{C}xy^2 zt^2 = 0$$
$$\mathbf{L}^{(1)}_2\mathbf{B}yzt - \mathbf{L}^{(2)}_2\mathbf{B}x^2 yzt + \mathbf{F}^{(1)}_2\mathbf{A}x^2 yt - \mathbf{F}^{(2)}_2\mathbf{A}x^2 yz^2 t + \mathbf{L}^{(3)}_2\mathbf{D}xyzt -$$
$$-\mathbf{L}^{(4)}_2\mathbf{D}xy^2 zt + \mathbf{F}^{(3)}_2\mathbf{C}xy^2 z - \mathbf{F}^{(4)}_2\mathbf{C}xy^2 zt^2 = 0$$
$$x = e^{-R_2 s}, \quad y = e^{-R_2\eta s}, \quad \upsilon = e^{-R_1 s}, \quad w = e^{-R_1\eta s}, \quad z = e_{13}, \quad t = e_{16}$$

where $T_j^{(i)}$ and $F_j^{(k)}$ are infinite matrices with elements $T_{npj}^{(i)}(s)$ and $F_{npj}^{(i)}(s)$, $M^{(i)}$, $N^{(i)}$ and $L_k^{(k)}$ are infinite diagonal matrices with elements $M_n^{(i)}(s)$, $N_n^{(i)}(s)$ and $L_{nj}^{(k)}(s)$ $(i=1\text{–}8;\ j=1,\ 2;\ k=1\text{–}4)$; $\mathbf{K}^{(l)}$ are infinite columns with elements $K_n^{(l)}(s)$ $(l=1,\ 2)$, and $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are infinite unknown columns with elements $A_n^L(s)$, $B_n^L(s)$, $C_n^L(s)$ and $D_n^L(s)$.

In the case of boundary conditions (2.1) and (2.3) (the axisymmetric surface load is specified on the inner sphere while the outer sphere is free from stresses), the elements of the matrices $\mathbf{T}_j^{(i)}$, $\mathbf{M}^{(i)}$, $\mathbf{N}^{(i)}$ and of the columns $\mathbf{K}^{(l)}$ have the form

$$T_{npj}^{(m)}(s)=Q_{nj}(-R_1s)C_{np}^{(m)}(s),\quad T_{npj}^{(2+m)}(s)=Q_{nj}(R_1s)C_{np}^{(m)}(s)\quad (j=1,2)$$

$$T_{np1}^{(4+m)}(s)=\eta^{-n}Q_{n2}(-R_1\eta s)S_{np}^{(m)}(s),\quad T_{np1}^{(6+m)}(s)=\eta^{-n}Q_{n2}(R_1\eta s)S_{np}^{(m)}(s)$$

$$T_{np2}^{(4+m)}(s)=\frac{Q_{n3}(-R_1\eta s)}{n(n+1)\eta^n}S_{np}^{(m)}(s),\quad T_{np2}^{(6+m)}(s)=\frac{Q_{n3}(-R_1\eta s)}{n(n+1)\eta^n}S_{np}^{(m)}(s)\quad (m=1,2)$$

$$M_n^{(1)}(s)=Q_{n1}(R_1s),\quad N_n^{(1)}(s)=\eta^{-n}n(n+1)Q_{n2}(R_1\eta s)$$

$$M_n^{(2)}(s)=Q_{n2}(R_1s),\quad N_n^{(2)}(s)=\eta^{-n}Q_{n3}(R_1\eta s)$$

$$K_n^{(1)}(s)=R_1^{n+3}s^np_{1n}^L(s),\quad K_n^{(2)}(s)=R_1^{n+3}s^nq_{1n}^L(s) \tag{2.11}$$

while the elements of the matrices $F_j^{(k)}$ and $L_j^{(k)}$ are given by

$$F_{npj}^{(m)}(s)=Q_{nj}(R_2s)C_{np}^{(m)}(s),\quad F_{np1}^{(2+m)}(s)=\eta^{-n}Q_{n2}(R_2\eta s)S_{np}^{(m)}(s)$$

$$F_{np2}^{(2+m)}(s)=n^{-1}(n+1)^{-1}\eta^{-n}Q_{n3}(R_2\eta s)S_{np}^{(m)}(s)\quad (m=1,2)$$

$$L_{nj}^{(1)}(s)=(-1)^nQ_{nj}(-R_2s),\quad L_{n1}^{(3)}(s)=n(n+1)(-\eta)^nQ_{n2}(-R_2\eta s)$$

$$L_{n2}^{(3)}(s)=(-\eta)^nQ_{n3}(-R_2\eta s),\quad L_{nj}^{(2k)}(s)=L_{nj}^{(2k-1)}(-s)\quad (j;k=1,2) \tag{2.12}$$

If, instead of boundary conditions (2.3), we consider the conditions (2.4), then, we must replace the following polynomials in relations (2.12): Q_{n1} by R_{n1}, Q_{n2} by R_{n0}, and Q_{n3} by R_{n3}. If we use conditions (2.2) instead of boundary conditions (2.1), then in (2.11) similar replacements are carried out, and the elements of the columns K_n have the form

$$K_n^{(1)}(s)=-R_1^{n+2}s^nU_n^L(s),\quad K_n^{(2)}(s)=-R_1^{n+2}s^nV_n^L(s) \tag{2.13}$$

Note that the elements of all these matrices and vectors are rational functions of the transformation parameter s.

In view of the length of the explicit formulae we will obtain a solution of system (2.10) for the special case of an acoustic medium.

3. AN ACOUSTIC MEDIUM

Passing to the limit as $\eta\to\infty$ $(\kappa\to 1)$ we obtain from (2.10) an infinite system of linear algebraic equations in the unknown coefficients $A_n^L(s)$ and $B_n^L(s)$ for an acoustic medium

$$\mathbf{M}\mathbf{A}\upsilon^2z+\mathbf{T}^{(1)}B-\mathbf{T}^{(2)}B\upsilon^2-\mathbf{T}^{(3)}\mathbf{B}z^2+\mathbf{T}^{(4)}\mathbf{B}\upsilon^2z^2=Z\upsilon z$$

$$\mathbf{L}^{(1)}Bz-\mathbf{L}^{(2)}Bx^2z+\mathbf{F}^{(1)}\mathbf{A}x^2-\mathbf{F}^{(2)}\mathbf{A}x^2z^2=0 \tag{3.1}$$

Here $\mathbf{T}^{(i)}$ and $\mathbf{F}^{(k)}$ are infinite matrices with elements $T_{np}^{(i)}(s)$ and $F_{np}^{(k)}(s)$, $\mathbf{M}$ and $\mathbf{L}^{(k)}$ are infinite diagonal matrices with elements $M_n(s)$ and $L_n^{(k)}(s)$ $(i=1-4;\ k=1,\ 2)$, and $\mathbf{Z}$ is an infinite column with elements $Z_n(s)$. In the case of different boundary conditions in accordance with (2.11) and (2.12) the elements of these matrices have the following form:

(a) an axisymmetric pressure $p_1(\tau,\ \theta_1)$ is applied to the surface of the cavity $r_1=R_1$

$$T_{np}^{(i)}(s)=T_{np1}^{(i)}(s)/(R_1 s)^2,\quad M_n(s)=M_n^{(1)}(s)/(R_1 s)^2,\quad Z_n(s)=K_n^{(1)}(s)/(R_1 s)^2$$

$$T_{np}^{(m)}(s)=R_{n0}(-R_1 s)C_{np}^{(m)}(s),\quad T_{np}^{(2+m)}(s)=R_{n0}(R_1 s)C_{np}^{(m)}(s)\quad (m=1,2)$$

$$M_n(s)=R_{n0}(R_1 s),\quad Z_n(s)=-R_n^{n+1}s^{n-2}p_{1n}^L(s)$$

(b) the velocity $V(\tau,\ \theta_1)$ is specified on the surface of the cavity $r_1=R_1$

$$T_{np}^{(m)}(s)=T_{np1}^{(m)}(s)=R_{n1}(-R_1 s)C_{np}^{(m)}(s)$$

$$T_{np}^{(2+m)}(s)=T_{np1}^{(2+m)}(s)=R_{n1}(R_1 s)C_{np}^{(m)}(s)\quad (m=1,2)$$

$$M_n(s)=M_n^{(1)}(s)=R_{n1}(R_1 s),\quad Z_n(s)=K_n^{(1)}(s)=-R_1^{n+2}s^n V_n^L(s)$$

(c) the velocity is zero on the external sphere $r_2=R_2$

$$F_{np}^{(m)}(s)=F_{np1}^{(m)}(s)=R_{n1}(R_2 s)C_{np}^{(m)}(s),\quad L_n^{(m)}(s)=L_{n1}^{(m)}(s)\quad (m=1,2)$$

$$L_n^{(1)}(s)=L_n^{(2)}(-s)=(-1)^n R_{n1}(-R_2 s)$$

The solution of the system of equations (3.1) can be represented in the form of series in exponents

$$\left\|\begin{matrix}\mathbf{A}\\ \mathbf{B}\end{matrix}\right\|=\sum_{i,k,n=0}^{\infty}\left(\left\|\begin{matrix}\mathbf{a}_{ikn}^{(1)}(s)\\ \mathbf{b}_{ikn}^{(1)}(s)\end{matrix}\right\|x^i\upsilon^{-k-1}z^n+\left\|\begin{matrix}\mathbf{a}_{ikn}^{(2)}(s)\\ \mathbf{b}_{ikn}^{(2)}(s)\end{matrix}\right\|x^i\upsilon^{-k-1}z^{-n-1}\right) \tag{3.2}$$

where $\mathbf{a}_{ikn}^{(l)}(s)$ and $\mathbf{b}_{ikn}^{(l)}(s)$ are infinite unknown vectors with corresponding elements $a_{ikn}^{(l,q)}(s)$ and $b_{ikn}^{(l,q)}(s)$ $(l=1,\ 2;\ q=0,\ 1,2,\dots)$.

Substituting series (3.2) into (3.1) and equating the coefficients of like powers of the variables x, υ and z we obtain recurrent relations for the coefficients $a_{ikn}^{(l,q)}(s)$, $b_{ikn}^{(l,q)}(s)$ $(l=1,\ 2;\ q=0,\ 1,2,\dots)$

$$\begin{aligned}
&a_{000}^{(1,q)}(s)=Z_q(s)/M_q(s),\quad a_{i0n}^{(1,q)}(s)=0\quad (i\geqslant 0,\ n\geqslant 0,\ in\neq 0)\\
&a_{i0n}^{(2,q)}(s)=b_{i0n}^{(l,q)}(s)=0,\quad b_{ikn}^{(l,q)}(s)=0\quad (i=0,1;\ k\geqslant 0,\ n\geqslant 0)\\
&b_{ik0}^{(1,q)}(s)=X_q(s)\left[b_{i-2,k0}^{(1,q)}(s)+\sum_{m=1}^{2}\sum_{p=0}^{\infty}(-1)^{q+m}C_{qp}^{(m)}(s)a_{i-2,k,2-m}^{(m,q)}(s)\right]\\
&b_{ik0}^{(2,q)}(s)=X_q(s)\left[b_{i-2,k0}^{(2,q)}(s)+\sum_{m=1}^{2}\sum_{p=0}^{\infty}(-1)^{q+m}C_{qp}^{(m)}(s)a_{i-2,k,m-1}^{(m,q)}(s)\right]\quad (i\geqslant 2,\ \ k\geqslant 0)\\
&b_{ikn}^{(1,q)}(s)=X_q(s)\left[b_{i-2,kn}^{(1,q)}(s)+\sum_{m=1}^{2}\sum_{p=0}^{\infty}(-1)^{q+m}C_{qp}^{(m)}(s)a_{i-2,k,n-2m+3}^{(m,q)}(s)\right]\\
&b_{ikn}^{(2,q)}(s)=X_q(s)\left[b_{i-2,kn}^{(2,q)}(s)+\sum_{m=1}^{2}\sum_{p=0}^{\infty}(-1)^{q+m}C_{qp}^{(m)}(s)a_{i-2,k,n+2m-3}^{(m,q)}(s)\right]\\
&(i\geqslant 2,\ \ k\geqslant 0,\ \ n\geqslant 1)
\end{aligned} \tag{3.3}$$

$$a_{ik0}^{(1,q)}(s)=\sum_{m=1}^{2}\sum_{p=0}^{\infty}(-1)^m C_{qp}^{(m)}(s)\left[Y_q(s)b_{i,k+2m-4,1}^{(1,q)}(s)-b_{i,k+2m-4,0}^{(2,q)}(s)\right]$$

$$a_{ik0}^{(2,q)}(s)=\sum_{m=1}^{2}\sum_{p=0}^{\infty}(-1)^m C_{qp}^{(m)}(s)\left[Y_q(s)b_{i,k+2m-4,0}^{(1,q)}(s)-b_{i,k+2m-4,1}^{(2,q)}(s)\right]$$

$(i\geq 0,\ \ k\geq 1)$

$$a_{ikn}^{(1,q)}(s)=\sum_{m=1}^{2}\sum_{p=0}^{\infty}(-1)^m C_{qp}^{(m)}(s)\left[Y_q(s)b_{i,k+2m-4,n+1}^{(1,q)}(s)-b_{i,k+2m-4,n-1}^{(1,q)}(s)\right]$$

$$a_{ikn}^{(2,q)}(s)=\sum_{m=1}^{2}\sum_{p=0}^{\infty}(-1)^m C_{qr}^{(m)}(s)\left[Y_q(s)b_{i,k+2m-4,n-1}^{(2,q)}(s)-b_{i,k+2m-4,n+1}^{(2,q)}(s)\right]$$

$(i\geq 0,\ \ k\geq 1,\ \ n\geq 1)$

Here it is formally assumed that $b_{1,-1,n}^{(l,q)}(s)=0$ $(i\geq 0,\ n\geq 0)$. The rational functions $X_q(s)$ and $Y_q(s)$ in (3.3) have the following form

$$X_q(s)=R_{q1}(R_2s)/R_{q1}(-R_2s),\quad Y_q(s)=M_q(-s)/M_q(s)$$

Passing to the limit as $\eta\to\infty$ $(\kappa\to 1)$ in expressions (2.9) for the coefficients $\sigma_{r_1r_1n}^{L}(r_1,s)$ and taking the solution (3.2) into account we obtain the following explicit formula for the coefficients $p_q^L(r_1,s)$ of the expansion of the pressure $p=-\sigma_{r_1r_1}$ in series in polynomials $P_q(\cos\theta_1)$

$$p_q^L(r_1,s)=\frac{1}{r_1^{q+1}}\sum_{i,k,n=0}^{\infty}\left\{\frac{R_{q0}(r_1s)}{s^{q-2}}\left[a_{ikn}^{(1,q)}(s)z^2+a_{ikn}^{(2,q)}(s)z^{-n-1}\right]x^i\upsilon^{-k-1}e_{11}+\right.$$
$$+\frac{G_{q0}(r_1s)}{s^{q-2}}\sum_{p=0}^{\infty}C_{qp}^{(1)}(s)[b_{ikn}^{(1,p)}(s)z^{n-1}+b_{ikn}^{(2,p)}(s)z^{-n-2}]x^i\upsilon^{-k-1}-$$
$$\left.-\frac{G_{q0}(r_1s)}{s^{q-2}}\sum_{p=0}^{\infty}C_{qp}^{(2)}(s)[b_{ikn}^{(1,p)}(s)z^{n+1}+b_{ikn}^{(2,p)}(s)z^{-n}]x^i\upsilon^{-k-1}\right\}\tag{3.4}$$

Hence, formulae (3.4) together with the recurrent relations (3.3) enable us to obtain a solution of the problem without using reduction to an infinite system (3.1). For a fixed number of terms in the series with respect to the angle θ_1 for the pressure, the coefficients in (3.4) are the product of rational and exponential functions of the parameter s. Their originals can be calculated in explicit form using appropriate theorems of the operational calculus.

Note that for an elastic medium the solution of system (2.12) can be represented in a form similar to (3.2)–(3.4), but the recurrent relations (3.3) have a more complex form.

4. EXAMPLE

As an example of the use of the above algorithm we will present the results of calculations for an acoustic medium with an internal cavity of radius $R_1=1$, on which a pressure of the form $p_1(\tau,\ \theta_1)=p_{00}H(\tau)$, is given, where $H(\tau)$ is the unit Heaviside function.

The velocity is zero on the external surface $R_2=2$. The offset of the centres of the sphere $\delta=0.5$.

The continuous curves in Fig. 1 are graphs of the change of the pressure with time in the medium, obtained taking four terms of the series in the angle θ_1 into account, at the following points $r_1=1.2$ and $\theta_1=\pi$ (curve 1), $r_1=1.4$ and $\theta_1=\pi$ (curve 2), and $r_1=1.5$ and $\theta_1=\pi$

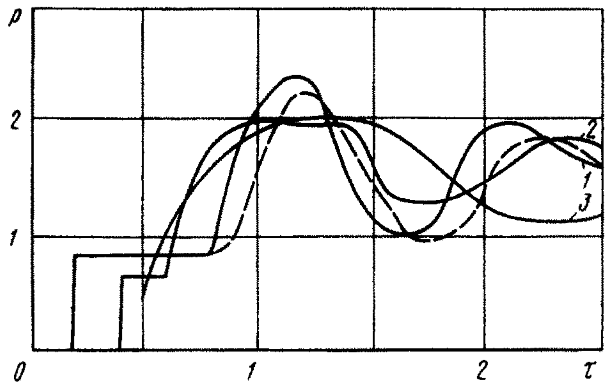

Fig. 1.

(curve 3). The dashed curves correspond to the results obtained by retaining the first three terms of the series in the angle θ_1 with parameters corresponding to curve 1. By comparing these curves it can be seen that the series converge fairly rapidly.

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